Equilibrio con hogares heterogeneos:

- Empresarios: $\theta_{ij} = \frac{1}{E}$, $i \in \{1, \ldots, E\}$. $\forall j$
- Trabajadores: $\theta_{ij} = 0$, $i \in \{E+1, \ldots, I\}$, $\forall j$.

$$n_t^*(w) = \frac{H}{1+\gamma}$$

$$n_e^*(w) = \frac{H}{1+\gamma} - \frac{\gamma}{1+\gamma} \frac{I}{E} \frac{\pi(w)}{w}$$

Supongamos que la solución es interior:

$$\boxed{\ell^* = \left(\frac{I}{J}\right) \frac{(1-\alpha)H}{1-\alpha+\gamma}} \quad (*) \qquad \boxed{L^* = \frac{I(1-\alpha)H}{1-\alpha+\gamma}} \quad (*)$$

$$L^* = N^* \quad \Rightarrow \quad \boxed{N^* = \frac{I(1-\alpha)H}{1-\alpha+\gamma}} \longrightarrow n_e^*, \; n_t^*$$

Camino 1: $\quad w^* = (1-\alpha)A\,\ell^{*-\alpha}$

$$\Rightarrow \boxed{w^* = (1-\alpha)A\left(\left(\frac{I}{J}\right)\frac{(1-\alpha)H}{1-\alpha+\gamma}\right)^{-\alpha}}$$

$$n_t^*(w) = \frac{H}{1+\gamma}$$

$$n_e^*(w) = \frac{H}{1+\gamma} - \frac{\gamma}{1+\gamma}\frac{I}{E}\frac{\pi(w)}{w} \qquad \Rightarrow \quad n_t^*, \; n_e^*.$$

Camino 2: $\quad N^* = E\,n_e^* + (I-E)\,n_t^*$

$$\frac{I(1-\alpha)H}{1-\alpha+\gamma} = E\,n_e^* + \frac{(I-E)H}{1+\gamma}$$

$$\epsilon n_e^* = \frac{I(1-\alpha)H}{1-\alpha+\gamma} - \frac{(I-\epsilon)H}{1+\gamma}$$

$$\Rightarrow \boxed{n_e^* = \frac{1}{\epsilon}\,\frac{I(1-\alpha)H}{1-\alpha+\gamma} - \frac{(I-\epsilon)}{\epsilon}\,\frac{H}{1+\gamma}}$$

Debemos verificar que efectivamente $n_e^* > 0$

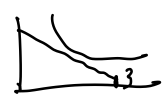
$$n_e^* > 0 \iff \frac{1}{\epsilon}\,\frac{I(1-\alpha)H}{1-\alpha+\gamma} - \frac{(I-\epsilon)}{\epsilon}\,\frac{H}{1+\gamma} > 0$$

$$\vdots$$

$$\iff \boxed{\frac{\epsilon}{I} > \frac{\gamma\alpha}{1-\alpha+\gamma}}$$

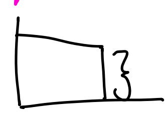
En equilibrio los empresarios trabajan sí y sólo si esta condición se cumple.

$\frac{\epsilon}{I}$: proporción de empresarios en la economía.



Si la proporción de empresarios es lo suficientemente grande $\Rightarrow$ los empresarios van a trabajar.

Si la proporción es suficientemente pequeña, empresarios NO trabajan en equilibrio.



Si $\frac{\epsilon}{I} < \frac{\gamma\alpha}{1-\alpha+\gamma} \Rightarrow \boxed{n_e^* = 0}$ $\quad \boxed{n_t^* = \frac{H}{1+\gamma}}$

$$\boxed{N^* = \epsilon \cancel{n_e^*}^{\,0} + (I-\epsilon)n_t^* = \frac{(I-\epsilon)H}{1+\gamma}} \qquad \boxed{L^* = \frac{(I-\epsilon)H}{1+\gamma}}$$

$$\boxed{\ell^* = \frac{(I-\epsilon)}{J}\,\frac{H}{1+\gamma}} \qquad \boxed{w^* = (1-\alpha)A\left(\frac{(I-\epsilon)}{J}\,\frac{H}{1+\gamma}\right)^{-\alpha}}$$

El equilibrio depende de los parámetros de la economía:

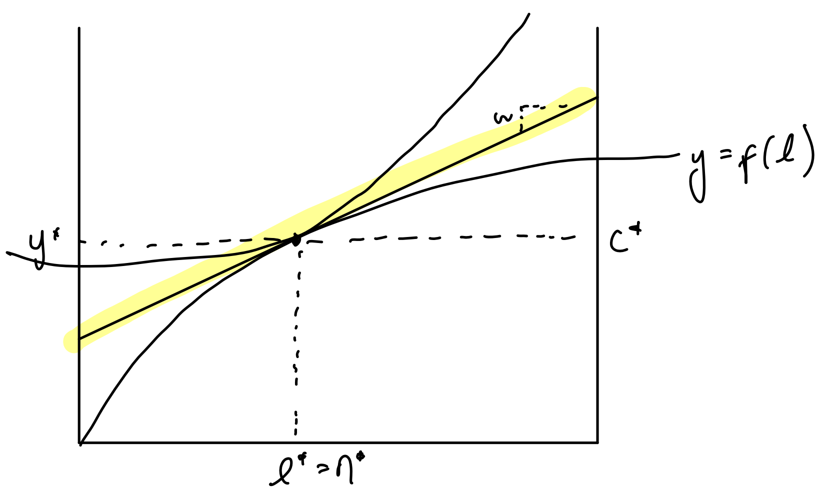
Si $\frac{\epsilon}{I} \geq \frac{\gamma\alpha}{1-\alpha+\gamma} \Rightarrow n_t^* = \frac{H}{1+\gamma}$, $n_e^* = \frac{H}{1+\gamma} - \frac{\gamma}{1+\gamma}\frac{\alpha}{1-\alpha+\gamma}\frac{I}{\epsilon}H$

...

Si $\frac{\epsilon}{I} < \frac{\gamma\alpha}{1-\alpha+\gamma} \Rightarrow n_t^* = \frac{H}{1+\gamma}$, $n_e^* = 0$

...

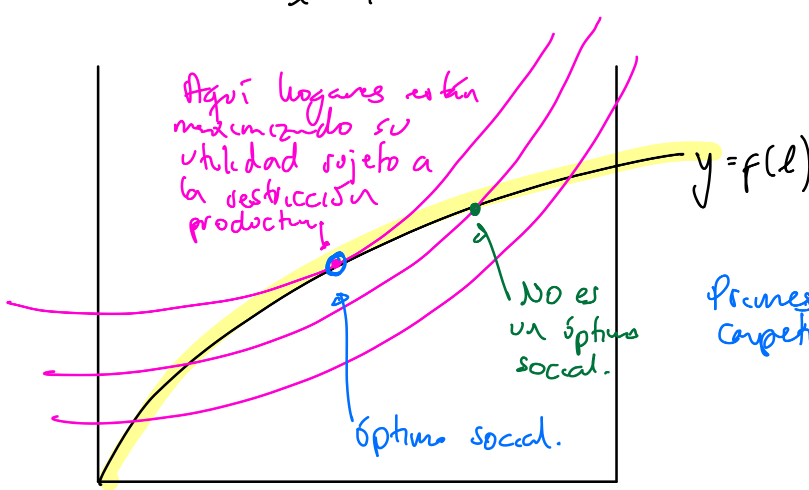
Maximización del bienestar social:



En equilibrio, w^* es tal que:

$c^* = y^*$

$\ell^* = n^*$



Primer teorema: un equilibrio competitivo es un óptimo social.

En óptimo social: $MRS_{(c^*,h^*)} = f'(\ell^*)$

$$\iff \boxed{\frac{\frac{\partial u(c^*,h^*)}{\partial h}}{\frac{\partial u(c^*,h^*)}{\partial c}} = f'(\ell^*)}$$

En eq. competitivo: $$\underbrace{\frac{\frac{\partial u(c^*,h^*)}{\partial h}}{\frac{\partial u(c^*,h^*)}{\partial c}}}_{\text{MRS}} = \underbrace{w}_{\text{pendiente de la restricción presupuestal.}}$$

$$\underbrace{f'(\ell^*)}_{\text{prod. marg. trabajo}} = \underbrace{w}_{\text{costo marg. del trabajo.}}$$

$$\Rightarrow \boxed{MRS_{(c^*,h^*)} = f'(\ell^*)}$$

Primer teorema del bienestar falla:

- Externalidades
- Bienes públicos
- impuestos distorsivos
- ⋮

Problema del planificador central:

$$\max_{c,h} \; u(c,h) \quad \text{s.a.} \quad h+\ell = H$$
$$c = f(\ell)$$

$$\iff \max_{\ell} \; u(f(\ell), H-\ell)$$

Derivando e igualando a cero:

$$\frac{\partial u}{\partial c}(\underbrace{f(\ell^*)}_{c^*}, \underbrace{H-\ell^*}_{h^*}) \frac{\partial f(\ell^*)}{\partial \ell} + \frac{\partial u}{\partial h}(f(\ell^*), H-\ell) \underbrace{\frac{\partial (\cancel{H}-\ell)}{\partial \ell}}_{-1} = 0$$

$$\frac{\partial u}{\partial c}(c^*, h^*) f'(\ell^*) - \frac{\partial u}{\partial h}(c^*, h^*) = 0$$

$$\Leftrightarrow \frac{\frac{\partial u}{\partial h}(c^*, h^*)}{\frac{\partial u}{\partial c}(c^*, h^*)} = f'(\ell^*)$$

Ej: $y = A\ell^{1-\alpha}$, $u(c,h) = \ln c + \gamma \ln h$.

Hay 1 individuo y 1 firma.

Problema del planificador central:

$$\max_{c,h} \ \ln c + \gamma \ln h \quad \text{s.a.} \quad H = h + \ell, \quad c = A\ell^{1-\alpha}$$

$$\Leftrightarrow \max_{\ell} \ \ln A\ell^{1-\alpha} + \gamma \ln (H-\ell)$$

Derivando e igualando a cero:

$$\frac{1}{A\ell^{1-\alpha}} \cdot A(1-\alpha)\ell^{-\alpha} - \frac{\gamma}{H-\ell} = 0$$

$$\vdots$$

$$\boxed{\ell^* = \frac{(1-\alpha)H}{1+\gamma-\alpha}}$$ — demanda laboral de eq.

Podemos "reconstruir" el equilibrio competitivo:

$n^* = \ell^* \Rightarrow \boxed{n^* = \frac{(1-\alpha)H}{1-\alpha+\gamma}}$

$$\boxed{w^* = (1-\alpha)A\left(\frac{(1-\alpha)H}{1-\alpha+\gamma}\right)^{-\alpha}}$$

$$\boxed{y^* = A\left(\frac{(1-\alpha)H}{1-\alpha+\gamma}\right)^{1-\alpha}} \qquad \boxed{c^* = A\left(\frac{(1-\alpha)H}{1-\alpha+\gamma}\right)^{1-\alpha}}$$

$\vdots$

Problema:

$$\max_{c,\ell} \; u(c, H-\ell) \quad \text{s.a.} \quad c = f(\ell)$$

$$\max_{c,\ell} \; \ln c + \gamma \ln(H-\ell) \quad \text{s.a.} \quad c = A\ell^{1-\alpha}$$

$$\mathcal{L} = \ln c + \gamma \ln(H-\ell) + \lambda\left(A\ell^{1-\alpha} - c\right)$$

$$\left.\begin{aligned} [c]&: \frac{1}{c} - \lambda = 0 \\ [\ell]&: \frac{-\gamma}{H-\ell} + \lambda(1-\alpha)A\ell^{-\alpha} = 0 \end{aligned}\right\} \quad \begin{aligned} 1/c &= \lambda \\ \frac{\gamma}{H-\ell} &= \lambda(1-\alpha)A\ell^{-\alpha} \end{aligned}$$

$$[\lambda]: A\ell^{1-\alpha} - c = 0$$

$$\Rightarrow \boxed{\frac{\gamma c}{H-\ell} = (1-\alpha)A\ell^{-\alpha}}$$

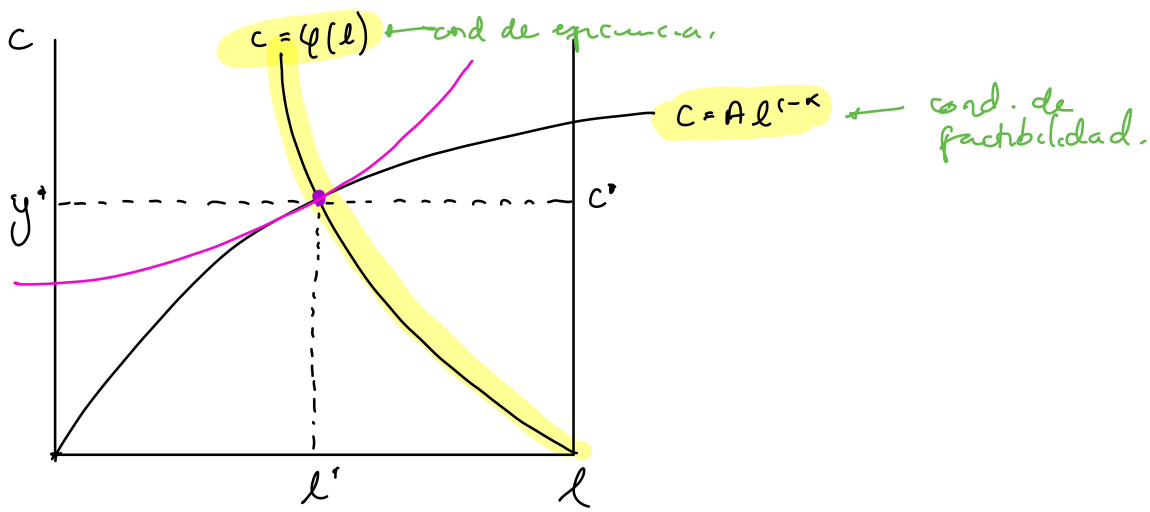
condición de eficiencia.

$$\boxed{c = A\ell^{1-\alpha}}$$ condición de factibilidad.

Cod de eficiencia:

$$c = \underbrace{\left(\frac{H-\ell}{\gamma}\right)(1-\alpha)A\ell^{-\alpha}}_{:= \varphi(\ell)}$$

Cond. de eficiencia: todos los puntos (c,ℓ) tal que $MRS(c,\ell)$ es igual a $f'(\ell)$


c
$c = \varphi(\ell)$
cond de eficiencia.
$c = A\ell^{1-\alpha}$
cond. de factibilidad.
y^*
c^*
ℓ^*
ℓ